Getting Students Started with Phytoplankton Modeling

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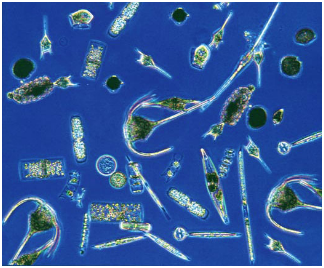
Project kept on:
http://www.jeanmarielinhart.info
Click on **Mathematical Modeling** (left column)
Click on **Phytoplankton Modeling**

Given at the Seventh International Symposium on Biomathematics and Ecology: Education and Research

Phytoplankton

- Phyto = plant, plankton = wanderer
- Base of the food chain
- Sequester CO_2



Credit http://www.seos-project.eu/modules/oceancurrents/images/mt001a5_rd01.jpg

Harmful aspects of phytoplankton

Harmful blooms can create a disgusting mess, kill fish, etc.

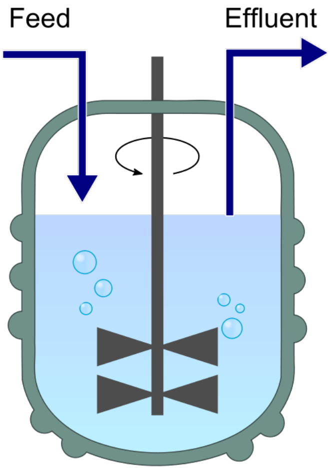
(All algae are plankton, but not plankton are algae...)

Credit http://toxics.usgs.gov/highlights/algal_toxins/

Purpose of project

- Get students familiar with the role of competition for resources in phytoplankton succession
  i.e. material from
  Ulrich Sommer, editor. *Plankton Ecology: Succession in Plankton Communities*. Springer-Verlag, 1989. Chapter 3
- Get students connecting mathematics with real life (biological) consequences.
- Get students a basic skill set so they can start doing research with phytoplankton models.

Chemostat Experimental setup



Credit: http://en.wikipedia.org/wiki/Chemostat

Mathematical Model

- $N_i(t)$ population density of phytoplankton, $i = 1, \ldots, n$
- $S_j(t)$ substrate (nutrient) concentration, $j = 1, \ldots, m$

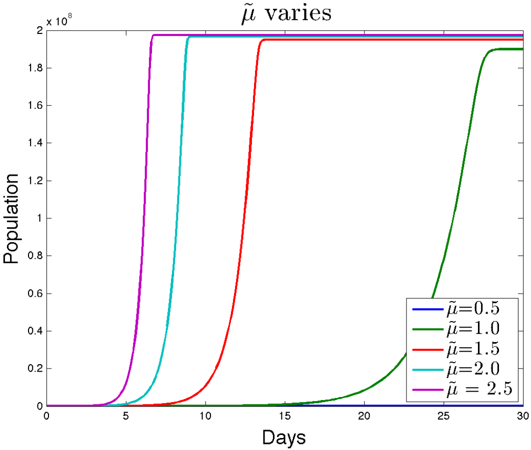
$$\frac{dN_i}{dt} = \mu_i N_i - \nu N_i$$
$$\frac{dS_j}{dt} = \nu(S_{\text{in}} - S_j) - \sum_{i=1}^{n} Q_i \mu_i N_i$$

Monod Relationship/Liebig's Law of the Minimum/Essential nutrient model:

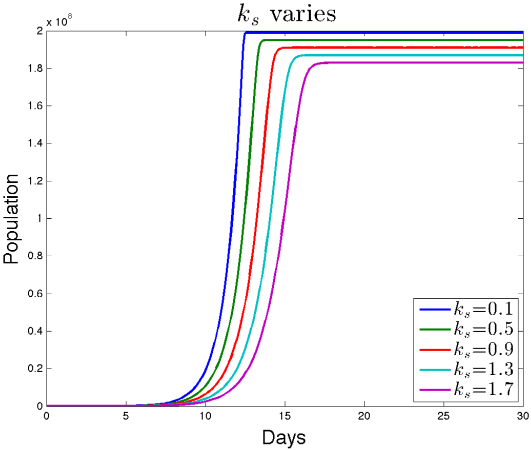
$$\mu_i = \tilde{\mu}_i \min_j \left\{ \frac{S_j}{S_j + k_{sj}} \right\}$$

Task 1: one species, one nutrient, sensitivity analysis

- Implement the model with given parameters.
- Systematically change input parameters to see how they affect the output

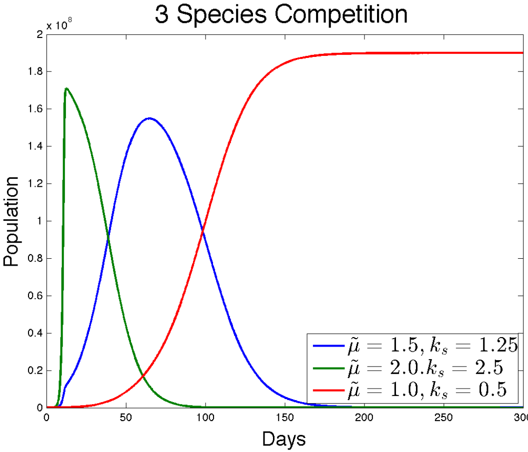


k_s sensitivity



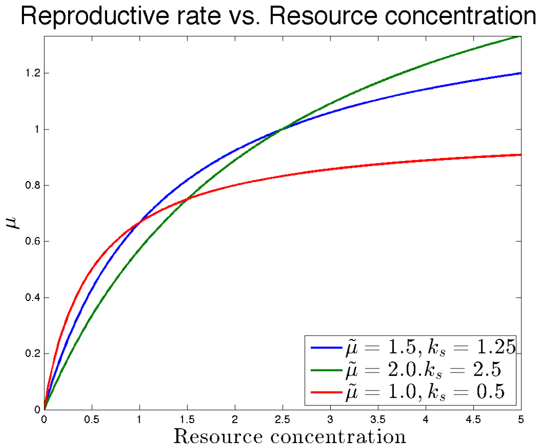
Task 2: Resource Saturation-Limitation Theory

- Implement the model with 3 species and one nutrient.
- Which species wins the competition and why?
- What does the succession pattern look like?
  Could we have predicted this without running the model?



Resource Saturation-Limitation Theory (2)

For each species we find $\mu_i = \dfrac{\tilde{\mu}_i S}{S + k_{si}}$ and plot



Task 2: Resource Saturation-Limitation Mathematics (3)

By the Monod Relationship:

$$\mu_i = \tilde{\mu}_i \frac{S}{S + k_{si}}$$

So growth with plentiful substrate is determined by $\tilde{\mu}_i$.

When resource limitation occurs, if a species survives $N \neq 0$:

$$\frac{dN_i}{dt} = 0 = \mu_i N - \nu N \quad \Longrightarrow \quad \mu_i = \nu$$

Solve for S to find the minimal amount of the substrate required for survival.

$$S = \frac{\nu k_{si}}{\tilde{\mu}_i - \nu}$$

Task 2: Resource Saturation-Limitation Theory (4)

- Using the Monod relationship determine the substrate concentrations at which coexistence is possible.

The minimal amount of the substrate required for survival is called R^*.

$$R^* = \frac{\nu k_{si}}{\tilde{\mu}_i - \nu}$$

- Coexistence is only possible with one substrate if R^* is minimal and equal for different species.
- This does not happen in this case.
- It is possible, with probability 0, e.g. $\nu = 0.5$, $\tilde{\mu}_1 = 2$, $k_{s1} = 2$; $\tilde{\mu}_2 = 1.25$, $k_{s2} = 0.5$.

Task 3: Optimal Resource Ratio

- Implement the model with one species and two substrates.
- Determine R^* (minimal nutrient required) for both substrates.
- Determine the optimal resource ratio for the population.

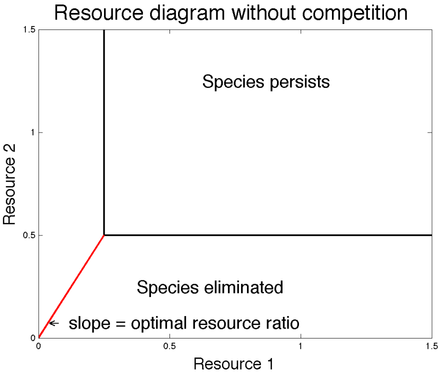
$\mu_i = \nu$ at steady state so for each species (i) and nutrient (j), we have to find R^* value:

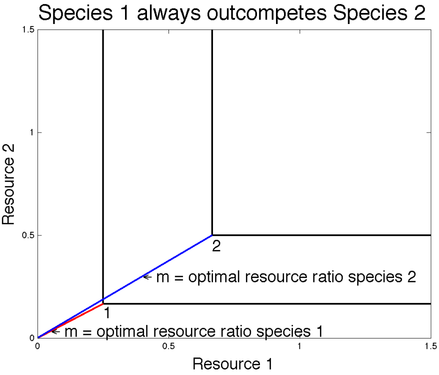
$$R^*_{ij} = \frac{\nu k_{sij}}{\tilde{\mu}_i - \nu}$$

Optimal Resource Ratio for species i: R^*_{i1}/R^*_{i2}

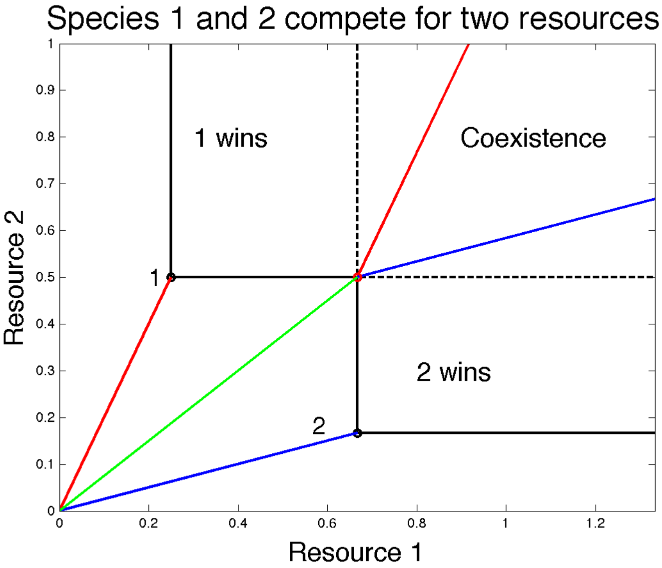
Zero Net Growth Isocline Diagrams

ZNGI $\longrightarrow$ reproductive rate $=$ flushing (death) rate



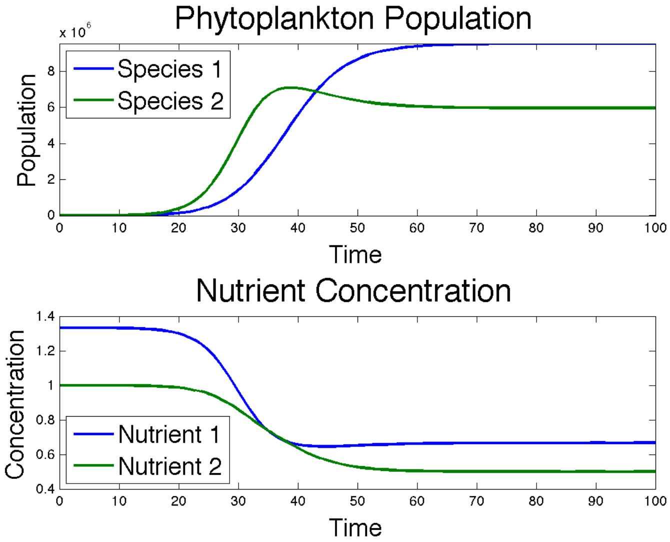


Competition and Coexistence Diagram



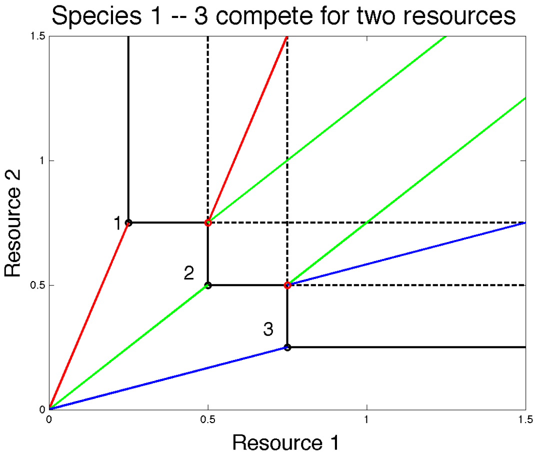
Capstone Activity for most students

Find substrate concentrations at which two species will coexist, and model coexistence.



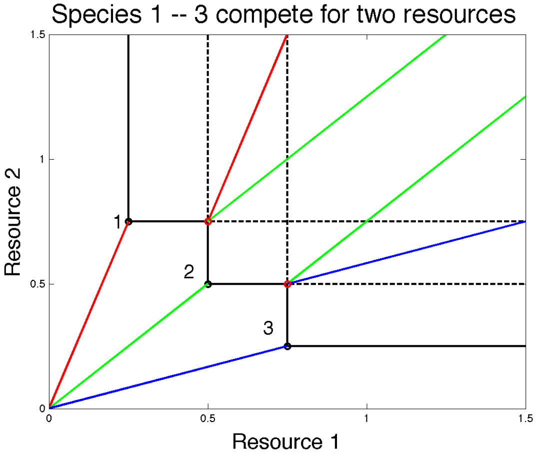
Task 4: Resource Ratio Theory

- Describe the succession dynamic and steady-state condition under varied substrate ratios in the source.



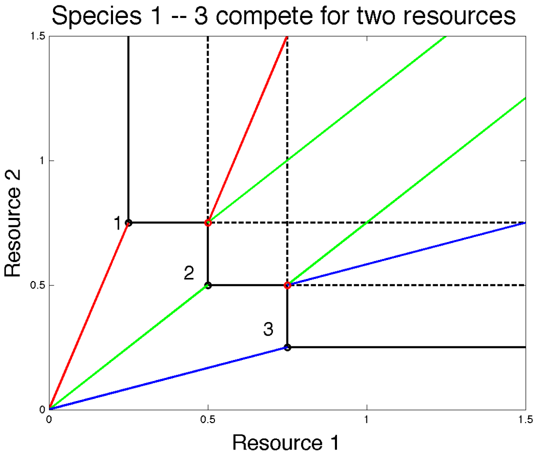
Task 5: Nutrient disturbances

- Vary the nutrient inflow rate. Increase or decrease the ratio of S_1 to S_2 as a periodic function.
- Experiment with longer and shorter periods. What happens?
- Can you find a middle value in which you see different species dominate but none become extinct?

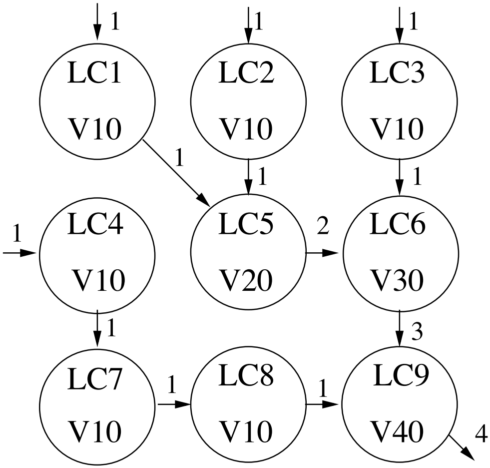


Task 6: Hydraulic disturbances

- Vary the flushing rate. Increase or decrease the flushing rate as a periodic function.
- Experiment with longer and shorter periods. What happens?
- Can you find a middle value in which you see different species dominate but none become extinct?



Current work: River/Reservoir model



Thanks for listening!