Let's talk about sets, baby.
Let's talk about one, two, three,
Let's talk about unions, intersections and sets that are empty.
Let's talk about sets!

Apologies to Salt N Pepa and all of their fans.

Section 6.1: Sets

Directions: Please fill in the (many) definitions from section 6.1 before coming to class, and do as many of the exercises as you can.

Example. Roster notation for a set simply lists the elements in the set. For example,

$$S = \{1, 2, 3, 4, 5\} \qquad \text{and} \qquad T = \{\text{cat, horse, dog, mouse}\}$$

are sets in roster notation.

Set-builder notation gives a rule for determining if you have a member of a set. For example,

$$U = \{x \,|\, 0 < x < 20 \text{ and } x \text{ is an even integer}\}$$

is another way of describing the set

$$U = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$$

Notice that 2 is an element of the set U above, and we write this as $2 \in U$. **What do you think $7 \notin U$ means?**

is an element of

7 is not an element of U

Exercise 1. Give at least two examples of sets using roster notation and two examples of sets using set-builder notation.

Set Builder

$S = \{x \mid x \text{ is a letter in mathematical}\}$

Roster

$S = \{m, a, t, h, e, i, c, l\}$

Set Builder

$T = \{x \mid 0 \le x \le 10\}$

Roster

$V = \{\text{apples, oranges, bananas, pineapples}\}$

Definition. set equality
Two sets are equal when they have exactly the same elements.

Exercise 2. Which of these sets are equal?
$A = \{\text{cat, dog, horse, mouse}\}$
$B = \{\text{cat, horse, dog}\}$
$C = \{\text{lion, zebra, flea, louse}\}$
$D = \{\text{cat, horse, dog, mouse}\}$

$A = D$

Definition. subset
If every element of a set A is also an element of a set B then we say that A is a subset of B and write $A \subseteq B$.

$A = \{1,2,3,4\}$ $\quad$ $\{1\} \subseteq A$ but $1 \in A$

$1 \subseteq A$ False $\quad$ Not the same

Note. If $A \subseteq B$, but $A \neq B$, then we can also write $A \subset B$, and we say that A is a **proper subset** of B.

$\{1,2\} \subset A$ True $\quad$ $\{1,2,3,4\} \subseteq A$

proper subset $\quad$ not proper

$\{1,2\} \subseteq A$ also true

Exercise 3. Which of the sets in exercise 2 are subsets of each other? Are any proper subsets?
$A = \{\text{cat, dog, horse, mouse}\}$
$B = \{\text{cat, horse, dog}\}$
$C = \{\text{lion, zebra, flea, louse}\}$
$D = \{\text{cat, horse, dog, mouse}\}$

$B \subset A$ proper subsets $\quad$ $B \subseteq A$

$D \subseteq A$ $\quad$ $A \subseteq D$ not proper

$D \subset A$ False

$B \subset D$ $\quad$ $B \subseteq D$

Definition. empty set, $\emptyset$
The set that contains no elements is called the **empty set** and is denoted as $\emptyset$.

$\phi \neq \{\phi\}$ $\quad$ $\{\}$ Empty set

no elements $\quad$ 1 element

Webassign has this wrong

$\phi = \{\}$

$\{\text{EMPTY}\}$ - webassign also wrong

Exercise 4. Find all of the subsets of $\{A, B, C\}$.

8 subsets $= 2^3 = 2^{\#\text{ elements}}$

$\{A\}, \{B\}, \{C\},$

$\{A,B\}, \{A,C\}, \{B,C\}, \{A,B,C\}, \phi$

$\phi \subset \{A,B,C\}$

Side note: elements vs. sets

Elements of $\{A,B,C\}$

A, B, C $\quad$ $A \in \{A,B,C\}$

$\{B\} \subset \{A,B,C\}$

Number of proper subsets:
all of the above except $\{A,B,C\}$
so $2^3 - 1 = 7$

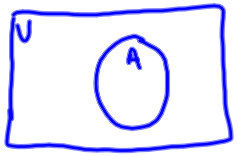
$2^{\#\text{ elements}} - 1 = \#$ proper subsets

Note. In contrast to the **empty set**, the **universal set** or **universe** is the set of all elements of interest to us in a given problem. For example, if we are interested in a problem about undergraduate students at TAMU, then our universal set might be all of the undergraduate students at TAMU, and our Math 141 class of students would be a proper subset of that universal set.

The universe contains everything.

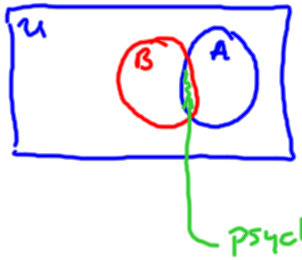
Exercise 5. Draw Venn Diagrams for each of the scenarios below where U, the universal set, is the set of all the undergraduate students at TAMU

a. A is the set of all the students taking Math 141 Section 506 in the Spring 2012 semester.



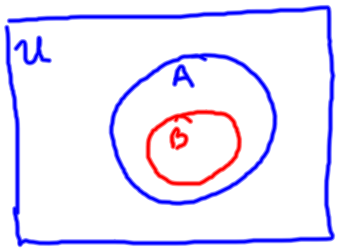
b. A is the set of all the students taking Math 141 Section 506 in the Spring 2012, and B is the set of all undergraduate psychology majors at TAMU.

There are psychology majors in Math 141



psych majors in Math 141

c. A is the set of all the students taking Math 141 Section 506 in the Spring 2012, and B is the set of all students in A who passed the first exam in Math 141 in Spring of 2012.



$B \subset A$

Definition. set union OR

Let A and B be sets. The **union** of two sets A and B is written $A \cup B$, and it is the set of all of the elements that belong to A or B or both.

In set-builder notation, we write

$$A \cup B = \{x \,|\, x \in A \text{ or } x \in B \text{ or both}\}$$

Definition. set intersection

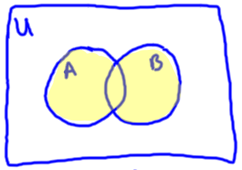
Let A and B be sets. The set of elements common to the sets A and B, $A \cap B$, is called the **intersection** of A and B.

AND

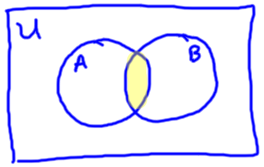
In set builder notation we write

$$A \cap B = \{x \,|\, x \in A \text{ and } x \in B\}$$

Exercise 6. Draw a Venn diagram shading $A \cup B$, and a second shading $A \cap B$.



$A \cup B$



$A \cap B$

Exercise 7. Consider the following sets
$A = \{\text{cat, dog, horse, mouse}\}$
$B = \{\text{cat, zebra, dog}\}$
$C = \{\text{lion, zebra, flea, louse}\}$

a. What is $A \cup B$?

$A \cup B = \{\text{cat, dog, horse, mouse, zebra}\}$

b. What is $A \cap B$?

$A \cap B = \{\text{cat, dog}\}$

c. What is $A \cup C$?

$A \cup C = \{\text{cat, dog, horse, mouse, lion, zebra, flea, louse}\}$

d. What is $A \cap C$?

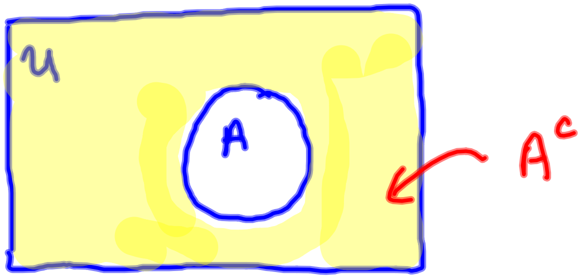
$A \cap C = \phi$ Disjoint Sets

Definition. complement of a set
If U is a universal set and A is a subset of U, then the set of all elements in U that are not in A is called the **complement** of A and is denoted A^c.

In set-builder notation

$$A^c = \{x \,|\, x \in U \text{ and } x \notin A\}$$

Exercise 8. Draw a Venn diagram of a universal set U, a set A, and shade the complement of A.



Exercise 9. If $U = \mathbb{Z} = \{x \,|\, x \text{ is an integer}\}$ and $A = \{x \,|\, x \text{ is an even integer}\}$ then what is A^c?

$A^c = \{x \mid x$ is an odd integer$\}$

Theorem 1. Set Complementation
If U is a universal set and A is a subset of U, then

a. $U^c = \emptyset$

b. $\emptyset^c = U$

c. $(A^c)^c = A$

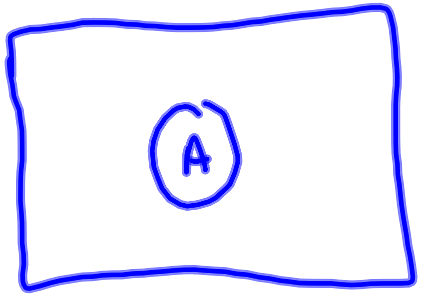
d. $A \cup A^c = U$

e. $A \cap A^c = \emptyset$

a) $\{x \mid x \in U \text{ and } x \notin U\} = \emptyset$

b) $\{x \mid x \in U \text{ and } x \notin \emptyset\} = U$

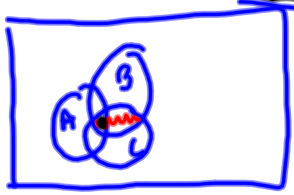
c)

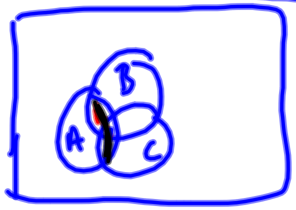


Proof.

Theorem 2. Properties of set operations
Let U be a universal set. If A, B and C are arbitrary subsets of U, then

- Commutative law for union: $A \cup B = B \cup A$
- Commutative law for intersection: $A \cap B = B \cap A$
- Associative law for union: $A \cup (B \cup C) = \cancel{(B \cup A) \cup}$ $(A \cup B) \cup (A \cup C)$
- Associative law for intersection: $A \cap (B \cap C) = (A \cap B) \cap C$
- Distributive law for union: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- Distributive law for intersection: $\boxed{A \cap (B \cap C) = (A \cap B) \cap (A \cap C)}$ something else

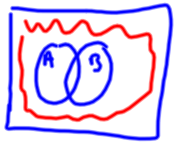




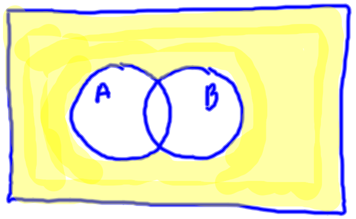
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Theorem 3. De Morgan's Laws
Let A and B be sets. Then

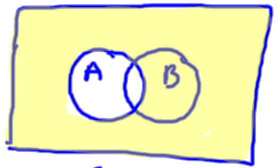
$$\begin{aligned}(A \cup B)^c &= A^c \cap B^c \\ (A \cap B)^c &= A^c \cup B^c\end{aligned}$$



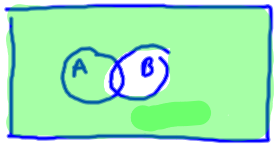
Proof. Show De Morgan's Laws using Venn Diagrams.



$(A \cup B)^c$ shaded



A^c

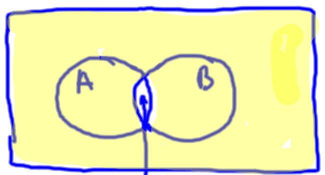


B^c

$A^c \cap B^c$ is shaded by both

$(A \cap B)^c = A^c \cup B^c$

$A^c \cup B^c$ is shaded by either

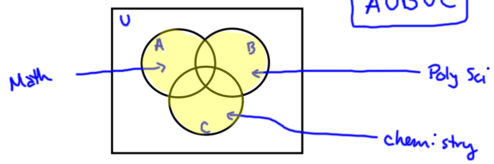


$A \cap B$ $\quad (A \cap B)^c$

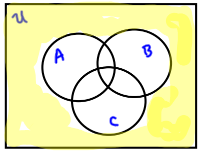
Exercise 10. Let U denote the set of all undergraduate students at TAMU and let
$A = \{x \in U \mid x \text{ is taking a math class}\}$ $B = \{x \in U \mid x \text{ is taking a political science class}\}$
$C = \{x \in U \mid x \text{ is taking a chemistry class}\}$
Find an expression in terms of A, B and C for each of the following sets, and draw a Venn Diagram shading the set.

a. The set of students taking at least one of the given classes.

$\boxed{A\cup B\cup C}$



b. The set of students taking at none of the given classes.

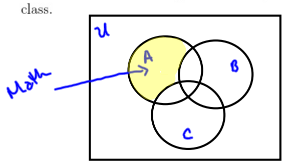


$(A\cup B\cup C)^c$

$A^c\cap B^c\cap C^c$

$A\cap(B^c\cup C^c)$

c. The set of students taking a math class, but not a political science or a chemistry class.



$\boxed{(B\cup C)^c\cap A}$

or

$\boxed{A\cap(B\cup C)^c}$

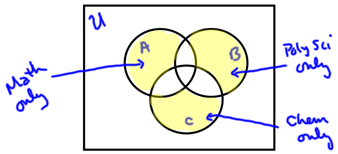
d. The set of students taking exactly one of the given classes.

intersection union

$[A\cap(B\cup C)^c]\cup$

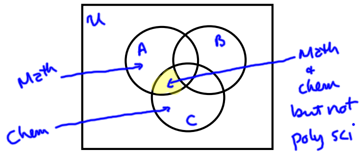
$[B\cap(A\cup C)^c]$

$\cup\,[C\cap(A\cup B)^c]$



e. The set of students taking both math and chemistry, but not political science.

$\boxed{(A\cap C)\cap B^c}$



Tip for success: Do the suggested homework on 6.1, especially 39, 45-55 odd, 63, and all of 67-76. This should go fast!

$\mathcal{U} = \{1, 2, 4, 6, 7, 9\}$

$A = \{1, 4, 7\}$ ✓

$B = \{2, 4, 6\}$ ✓

What is $A \cup B$?

$\{1, 4, 7, 2, 6\}$

union = joint means or
in A or in B

What is $A \cap B$?

$\{4\}$

intersection = stuff in common
means and
in A and in B

What is $A^c \cap B$?

$A^c = \{2, 6, 9\}$

A^c = in $\mathcal{U}$ but not in A

$A^c \cap B = \{2, 6\}$