

Let's talk about sets, baby.
 Let's talk about one, two, three,
 Let's talk about unions, intersections and sets that are empty.
 Let's talk about sets!

Apologies to Salt N Pepa and all of their fans.

Section 6.1: Sets

Directions: Please fill in the (many) definitions from section 6.1 before coming to class, and do as many of the exercises as you can.

Example. Roster notation for a set simply lists the elements in the set. For example,

$$S = \{1, 2, 3, 4, 5\} \quad \text{and} \quad T = \{\text{cat, horse, dog, mouse}\}$$

are sets in roster notation.

Set-builder notation gives a rule for determining if you have a member of a set. For example,

$$U = \{x \mid 0 < x < 20 \text{ and } x \text{ is an even integer}\}$$

is another way of describing the set

$$U = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$$

Notice that 2 is an element of the set U above, and we write this as $2 \in U$. **What do you think $7 \notin U$ means?**

Exercise 1. Give at least two examples of sets using roster notation and two examples of sets using set-builder notation.

Definition. set equality

Exercise 2. Which of these sets are equal?

$$A = \{\text{cat, dog, horse, mouse}\}$$

$$B = \{\text{cat, horse, dog}\}$$

$$C = \{\text{lion, zebra, flea, louse}\}$$

$$D = \{\text{cat, horse, dog, mouse}\}$$

Definition. subset

Note. If $A \subseteq B$, but $A \neq B$, then we can also write $A \subset B$, and we say that A is a **proper subset** of B .

Exercise 3. Which of the sets in exercise 2 are subsets of each other? Are any proper subsets?

Definition. empty set, \emptyset

Exercise 4. Find all of the subsets of $\{A, B, C\}$.

Note. In contrast to the **empty set**, the **universal set** or **universe** is the set of all elements of interest to us in a given problem. For example, if we are interested in a problem about undergraduate students at TAMU, then our universal set might be all of the undergraduate students at TAMU, and our Math 141 class of students would be a proper subset of that universal set.

Exercise 5. Draw Venn Diagrams for each of the scenarios below where U , the universal set, is the set of all the undergraduate students at TAMU

- a. A is the set of all the students taking Math 141 Section 506 in the Spring 2012 semester.

- b. A is the set of all the students taking Math 141 Section 506 in the Spring 2012, and B is the set of all undergraduate psychology majors at TAMU.

- c. A is the set of all the students taking Math 141 Section 506 in the Spring 2012, and B is the set of all students in A who passed the first exam in Math 141 in Spring of 2012.

Definition. set union

In set-builder notation, we write

$$A \cup B = \{x \mid x \in A \text{ or } x \in B \text{ or both}\}$$

Definition. set intersection

In set builder notation we write

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

Exercise 6. Draw a Venn diagram shading $A \cup B$, and a second shading $A \cap B$.

Exercise 7. Consider the following sets

$$A = \{\text{cat, dog, horse, mouse}\}$$

$$B = \{\text{cat, zebra, dog}\}$$

$$C = \{\text{lion, zebra, flea, louse}\}$$

a. What is $A \cup B$?

b. What is $A \cap B$?

c. What is $A \cup C$?

d. What is $A \cap C$?

Definition. complement of a set

In set-builder notation

$$A^c = \{x \mid x \in U \text{ and } x \notin A\}$$

Exercise 8. Draw a Venn diagram of a universal set U , a set A , and shade the complement of A .

Exercise 9. If $U = \mathbb{Z} = \{x \mid x \text{ is an integer}\}$ and $A = \{x \mid x \text{ is an even integer}\}$ then what is A^c ?

Theorem 1. Set Complementation

Proof.

□

Theorem 2. Properties of set operations

Let U be a universal set. If A , B and C are arbitrary subsets of U , then

Theorem 3. De Morgan's Laws

Let A and B be sets. Then

Proof. Show De Morgan's Laws using Venn Diagrams.

□

Exercise 10. Let U denote the set of all undergraduate students at TAMU and let
 $A = \{x \in U \mid x \text{ is taking a math class}\}$ $B = \{x \in U \mid x \text{ is taking a political science class}\}$
 $C = \{x \in U \mid x \text{ is taking a chemistry class}\}$

Find an expression in terms of A , B and C for each of the following sets, and draw a Venn Diagram shading the set.

a. The set of students taking at least one of the given classes.

b. The set of students taking at none of the given classes.

c. The set of students taking a math class, but not a political science or a chemistry class.

d. The set of students taking exactly one of the given classes.

e. The set of students taking both math and chemistry, but not political science.

Tip for success: Do the suggested homework on 6.1, especially 39, 45-55 odd, 63, and all of 67-76. This should go fast!